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S. Y. B. Sc. (Sem. - III) Examination

March - 2023

Mathematics : MTH - 301 : Paper - V

Time: 1 Hour] **[Total Marks: 50**

સૂચના : / Instructions

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **S. Y. B. Sc. (Sem. - III)**

Name of the Subject :

☛ **Mathematics : MTH - 301 : Paper - V**

Subject Code No.: **2103000203023001**

Seat No.:

Student's Signature

(2) All questions are compulsory.
(3) Section I contains 16 questions of 1 mark each.
(4) Section II contains 17 questions of 2 marks each.
(5) Use of non-programmable scientific calculator is allowed.

***O.M.R. Sheet* ભરવા અંગેની અગત્યની સૂચનાઓ આપેલ**
***O.M.R. Sheet*ની પાછળ છાપેલ છે.**
Important instructions to fillup O.M.R. Sheet
are given on back side of the provided O.M.R. Sheet.

Section - I

Choose the correct option: 01 marks **[16]**

Q. (1) If $x^2 + y^2 = r^2$ and $\tan\theta = \frac{y}{x}$ then the value of $\frac{\partial(r, \theta)}{\partial(x, y)}$ is equal to _____.

a) $\sin\theta$ b) $\cos\theta$

c) r d) $\frac{1}{r}$

Q. (2) Transformation equations connecting Cartesian coordinates and polar coordinators are _____.

a) $x = r\sin\theta, y = r$ b) $x = r\cos\theta, y = r$

c) $x = r\sin\theta, y = r\cos\theta$ d) $x = r\cos\theta, y = r\sin\theta$

Q. (3) Under usual notations if $f_x(a, b) = 0 = f_y(a, b)$, $r < 0$, $rt - s^2 > 0$ then $f(a, b)$ is _____.

a) Maximum value of f

b) Minimum value of f

c) Neither maximum nor minimum

d) Maximum as well as minimum of f

Q. (4) Minimum value of $f(x, y) = x^3 + y^3 - 3x - 12y + 5$ is _____.

a) –12 b) 5

c) –13 d) 23

Q. (5) The extreme point of the function $u = x^3 + y^3 - 3axy$ is _____.

a) $(a, 0)$ b) $(-a, 0)$

c) $(0, 0)$ d) (a, a)

Q. (6) Possible extreme point of the function $u = x^2 + 2y^2 - x$ is _____.

a) $(0, \frac{1}{3})$ b) $(\frac{1}{2}, 0)$

c) $(0, \frac{1}{2})$ d) $(\frac{1}{2}, \frac{1}{3})$

Q. (7) $\nabla(f - g)$ is equal to _____.

a) $\nabla f + \nabla g$ b) $\nabla f - \nabla g$

c) ∇f d) ∇g

Q. (8) Under usual notations div(curl $\vec{f}$) is equal to _____.

a) 1 b) 0

c) 2 d) –1

Q. (9) If $f(x, y, z) = 3x^2 - y^3z^2$ then grad f at the point (1, –2, –1) is _____.

a) $-12\vec{i} - 9\vec{j} - 16\vec{k}$ b) $12\vec{i} - 9\vec{j} - 16\vec{k}$

c) $-12\vec{i} + 9\vec{j} - 16\vec{k}$ d) $-12\vec{i} - 9\vec{j} + 16\vec{k}$

Q. (10) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then $\nabla f(r)$ is equal to _____.

a) 0 b) $\hat{r}$

c) $f'(r)\nabla r$ d) 1

Q. (11) A vector field A is conservative if and only if $\nabla \times A$ is equal to _____.

a) 1 b) 0

c) 2 d) –1

Q. (12) The value of $\lim\limits_{(x, y)\to(1,1)} \dfrac{x^2+y^2}{x+y}$ is _____.

a) 1 b) 2

c) $\frac{1}{2}$ d) $\frac{1}{4}$

Q. (13) The value of f_y at the point $(x, y) = (2, -1)$ where

$$f(x, y) = \begin{cases} \dfrac{x-y}{x+y}, & x+y \neq 0 \\ 0, & x+y = 0 \end{cases}$$ is _____.

a) –4 b) –2

c) –3 d) –1

Q. (14) The value of f_{xy} where $f(x, y) = e^x \sin y + e^y \sin x$ is _____.

a) $e^x \sin y + e^y \sin x$ b) $e^x \cos y + e^y \sin x$

c) $e^x \sin y + e^y \cos x$ d) $e^x \cos y + e^y \cos x$

Q. (15) If $z = f(x, y)$ is differentiate function and x, y are functions of t then $\frac{dz}{dt}$ is equal to _____.

a) $\frac{dx}{dt} + \frac{dy}{dt}$ b) $\frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial x}\frac{dy}{dt}$

c) $\frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$ d) $\frac{\partial z}{\partial y}\frac{dx}{dt} + \frac{\partial z}{\partial x}\frac{dy}{dt}$

Q. (16) The degree of the homogeneous function $f(x, y) = x^3 + y^3$ is _____.

a) 4 b) 2

c) 3 d) 1

Section - II

Choose the correct option: 02 Marks **[34]**

Q. (17) The value of $\nabla\left(\frac{f}{g}\right)$; $g \neq 0$, is equal to _____.

a) $\frac{g\nabla f - f\nabla g}{g^2}$ b) $\frac{\nabla f - \nabla g}{g^2}$

c) $\frac{f\nabla f - g\nabla g}{g^2}$ d) $\frac{1}{g^2}$

Q. (18) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then $\nabla f(r) \times \vec{r}$ is equal to _____.

a) $\vec{0}$ b) $\hat{r}$

c) $f'(r)\nabla r$ d) 1

Q. (19) If $\vec{u} = xyz\vec{i} + xz^2\vec{j} - y^3\vec{k}$ then $\frac{\partial^2 \vec{u}}{\partial z^2}$ at (1, 1, 0) is equal to _____.

a) $-2\vec{j}$ b) $-2\vec{i}$

c) $2\vec{k}$ d) $2\vec{j}$

Q. (20) The gradient of $f(x, y, z) = x^2y + yz + xz$ is equal to _____.

a) 0

b) $(2xy + z)\ \vec{i} + (x^2 + z)\ \vec{j} + (x + y)\ \vec{k}$

c) 1

d) $xy\ \vec{i} + yz\ \vec{j} + x\ \vec{k}$

Q. (21) The gradient of $f(x, y) = x^2 + y$ is equal to _____.

a) 0

b) $2x\ \vec{i} + \vec{j}$

c) 1

d) $xy\ \vec{i} + y\ \vec{j}$

Q. (22) The curl of $\vec{u} = x^2y\ \vec{i} + xy\ \vec{j} + yz\ \vec{k}$ at (0, 0, 1) is equal to _____.

a) 0

b) $\vec{j}$

c) 1

d) $\vec{i}$

Q. (23) If $\vec{r} = (1 + sint)\,\hat{i} + (1 - cost)\,\hat{j} + 3t^2\,\hat{K}$, then velocity $\vec{v}$ = _____.

a) $cos\ t\ \hat{i} - cos\ t\ \hat{j} + 6t\ \hat{K}$

b) $cos\ t\ \hat{i} + sin\ t\ \hat{j} + 6\ t\ \hat{K}$

c) $-sin\ t\ \hat{i} + cos\ t\ \hat{j} + 6t\ \hat{K}$

d) $cos\ t\ \hat{i} - sin\ t\ \hat{j} + 6\ \hat{K}$

Q. (24) If $\overline{u}$ and $\overline{v}$ are two vector functions then $div(\overline{u} \times \overline{v})$ = _____.

a) $\overline{v}.\ curl\ (\overline{u}) - \overline{u}.\ curl\ (\overline{v})$

b) $\overline{v}.\ \text{curl}(\overline{v}) - \overline{u}.\ curl(\overline{u})$

c) $\overline{u}.\ curl\ (\overline{u}) - \overline{v}.\ curl\ (\overline{v})$

d) $\overline{u}.\ \text{curl}\ (\overline{v}) - \overline{v}.\ curl\ (\overline{u})$

Q. (25) If $\vec{F} = 3xy\ \hat{i} - y^2\ \hat{j}$ then $\int_C \vec{F}\,.d\vec{r}$ = _____, where C is the curve, in the xy plane, $y = 2x^2$ from (0, 0) to (1, 2).

a) –7/6

b) –6/7

c) 7/6

d) 6/7

Q. (26) If $f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} ; & x \neq 0, y \neq 0 \\ 0 \quad ; & x = 0, y = 0 \end{cases}$ then

$\lim\limits_{(x, y)\to(0, 0)} f(x, y) =$ _____.

a) 1 b) 0

c) 2 d) Does not exist

Q. (27) $\lim\limits_{(x, y)\to(1, 2)} (xy - 3x + 4)$ is equal to _____.

a) 1 b) 0

c) 2 d) 3

Q. (28) $\lim\limits_{(x, y)\to(0, 0)} \frac{\tan(x+y)}{x+y}$ is equal to _____.

a) 1 b) 0

c) 2 d) Does not exist

Q. (29) If $f(x, y) = \sin^{-1}\left(\frac{x}{y}\right)$; $|y| \neq 0$ then f_y is equal to _____.

a) $\frac{-x}{x^2-y^2}$ b) $\frac{-x}{y^2-x^2}$

c) $\frac{-x}{y\sqrt{y^2-x^2}}$ d) $\frac{-xy}{\sqrt{x^2-y^2}}$

Q. (30) If $f(x, y) = x \cos y + y \cos x$ then f_{xx} is equal to _____.

a) $y \cos x$ b) $-y \cos x$

c) y d) 0

Q. (31) If $x = \rho \cos \varphi$, $y = \rho \sin \varphi$, $z = z$ then $\frac{\partial(x, y, z)}{\partial(\rho, \varphi, z)}$ is equal to _____.

a) ρ b) 1

c) φ d) z

Q. (32) Which of the following is an extreme point for the function $f(x, y) = 2(x - y)^2 - x^4 - y^4$?

a) $(1, 1)$ b) $(2, 0)$

c) $(\sqrt{2}, -\sqrt{2})$ d) $(1, 2)$

Q .(33) The extreme value of $f(x, y) = 6x^2 - 20x + 12y^2 + 4y$ is _____.

a) 17 b) 18

c) –16 d) –17

SPACE FOR ROUGH WORK